A SQUARE FUNCTION ESTIMATE FOR THE SCHRÖDINGER EQUATION WITH APPLICATIONS

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We will consider the Schrödinger equation, $i\partial_t u + \Delta u = 0$, in $\mathbb{R}^{d+1}$, with initial data u_0, and S defined by

$$Su_0 = \Big(\int_0^1 |u(\,\cdot\,,t)|^2 dt \Big)^{1/2},$$

and prove that $S : L^p_\alpha(\mathbb{R}^d) \to L^p(\mathbb{R}^d)$ is bounded when $2 + 4/d < p < \infty$ and $\alpha = 2d(1/2 - 1/p) - 1$. Here, $L^p_\alpha(\mathbb{R}^d)$ denotes the L^p-Sobolev space with α derivatives, and the estimate is sharp with respect to α. We will also see how this yields the Fefferman–Stein–Miyachi fixed-time estimates, and the best known bounds for Bochner–Riesz in low dimensions. This is joint work with Sanghyuk Lee and Andreas Seeger.

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